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Corrigendum

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Volume 67, Number 2 (1991), in the article "The n-Width of the Unit Ball of H^q," by S. D. Fisher and M. I. Stessin, pages 347–356: Several people have kindly pointed out to us that the proof of the upper bound in our article for the linear n-width of the unit ball of H^q in L^p is incorrect. This corrigendum provides a correct proof in two important cases: when $p \le q$ or when $q = 2$ (and p is any number.) Thus, the equalities stated are proved for the case $p \le q$ and for the case when $p > q = 2$ and the hyperbolic radius r_0 of **E** satisfies $0 < r_0 < (1 - \cos(\pi/p))/\sin(\pi/p)) = \tan(\pi/2p)$.

First suppose that $p \le q$. Let B be a Blaschke product of degree r, $r \le n$, with zeros at $\{z_1, \dots, z_r\}$, and let g be the (unique) normalized solution of (1) for the measure $|B|^p \, d\mu$. Define T: $H^q \to L^p$ by

$$Tf(z) = B(z)g(z)\int \left[f(e^{i\theta})/B(e^{i\theta})g(e^{i\theta})\right] P(z; e^{i\theta}) \, d\theta,$$

where $P(z; e^{i\theta})$ is the Poisson kernel for z. It is easy to verify that $Tf = f$ when f vanishes at the zeros of B. In general, let $\phi_1, \dots, \phi_r$ be bounded analytic functions on Δ such that $\phi_i(z_j) = \delta_{ij}$. Then $f = f - \sum_1^r f(z_j)\phi_j + \sum_1^r f(z_j)\phi_j = g + \sum_1^r f(z_j)\phi_j$, where g vanishes at $z_1, \dots, z_r$. Hence,

$$Tf = Tg + T\left(\sum_1^r f(z_j)\phi_j\right) = f - \sum_1^r f(z_j)\phi_j + \sum_1^r f(z_j)T\phi_j$$

$$= f - \sum_1^r f(z_j)(\phi_j - T\phi_j),$$

which shows that $T = I - P$, where P is linear of rank $r \le n$. Thus, the linear n-width of the unit ball of H^q in L^p is certainly bounded above by the norm of T. However, for f in the unit ball of H^q, we have

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$$\int_E \left| B(z)g(z)\int_0^{2\pi}[f(e^{i\theta})/B(e^{i\theta})g(e^{i\theta})]P(z;e^{i\theta})\,d\theta\right|^p d\mu(z)$$

$$\leq \int_0^{2\pi}|f(e^{i\theta})/g(e^{i\theta})|^p \int_E |B(z)g(z)|^p P(z;e^{i\theta})\,d\mu(z)\,d\theta$$

$$= \delta^p \int_0^{2\pi}|f(e^{i\theta})/g(e^{i\theta})|^p |g(e^{i\theta})|^q\,d\theta$$

$$= \delta^p \int_0^{2\pi}|f(e^{i\theta})|^p |g(e^{i\theta})|^{q-p}\,d\theta$$

$$\leq \delta^p = \sup\left\{\int_E |B(z)h(z)|^p\,d\mu(z) : h \in A_q\right\}$$

$$= \sup\left\{(\|f\|_p)^p : f \in A_q \text{ and } f(z_k) = 0,\ k = 1,\dots,r\right\}.$$

Because B is an arbitrary Blaschke product of degree n or less, this establishes that the linear n-width is no more than

$$\inf\left[\sup\{\|f\|_p : f \in A_q \text{ and } f(z_k) = 0,\ k = 1,\dots,r\} : z_1,\dots,z_r \in \Delta,\ r \leq n\right].$$

However, this number is already known to be the lower bound for both the Gelfand and the Kolmogorov n-widths, so that equality is established.

The case $q = 2$ is easier. Let $z_1,\dots,z_n$ be (distinct) points of Δ, let $f \in A_2$, and let T be the orthogonal projection of H^2 onto the subspace of H^2 functions which vanish at the zeros of B. Then $I - T$ has rank n and T has operator norm one so that $Tf = Bh$, where h has H^2 norm at most one. Therefore, $\|Tf\|_p = \|Bh\|_p \leq \sup\{\|f\|_p : f \in A_2 \text{ and } f(z_k) = 0, k = 1,\dots,n\}$. Since B is an arbitrary Blaschke product, this shows that the linear n-width is no more than

$$\inf\left[\sup\{\|f\|_p : f \in A_2 \text{ and } f(z_k) = 0,\ k = 1,\dots,r\} : z_1,\dots,z_n \in \Delta\right]$$

and we are done.

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